**TO TEST OR NOT TO TEST: PRELIMINARY ASSESSMENT OF NORMALITY WHEN COMPARING TWO INDEPENDENT SAMPLES**

It is very important to ensure that data meets the normality conditions before performing main test of the two-sample t test. This requirement, if not met, can lead to incorrect conclusions in the main test results. Many publications, especially in the medical field, have been cited for errors due to violation of distributional assumptions.

One widely accepted approach is by means of Shapiro-Wilk test pretest. If the pretest result is statistically insignificant, then the main t test is adopted otherwise the nonparametric equivalent Mann-Whitney U test is used.

This method also has several drawbacks. First, the fact that the pretest result is insignificant does not mean that the data is normally distributed. Because insufficient evidence exists against the null hypothesis, normality will be considered true, which is flawed. People also argue that preliminary testing is about assumptions of the population and not the samples. Most times samples are not representative of the population from which they were drawn from. Actual test may still be robust to violations of the assumptions and so pretesting may be irrelevant.

Most importantly, because preliminary tests are usually applied to the same data as the subsequent test, it leads to uncontrollable Type I error rates. In other words, it alters Type I error rates.

The main objective of this article is to assess the impact of preliminary test for normality on the results of the two-sample t test. A simulation study was conducted on equally sized samples for two groups in two selection strategies. For one strategy, the two-sample t test was conducted if both samples had passed the preliminary test for normality, otherwise the nonparametric Mann-Whitney U test was performed. In the other strategy, the two samples t test was conducted if the residuals of the collapsed samples had passed preliminary test for normality. Sample sizes 10, 20, 30, 40, and 50 were generated from exponential, uniform, and normal distributions. The procedure was repeated until 10,000 pairs of samples had passed the preliminary screening for normality (either strategy I or II) with 0.100, 0.05, 0.01, 0.005 or no pretest and the t test is performed. Similarly, the procedure was repeated until 10,000 pairs of samples had failed the preliminary screening for normality (either strategy I or II) with 0.100, 0.05, 0.01, 0.005 or no pretest and the Mann-Whitney U test is performed. Lastly, 100,000 pairs of samples were generated to assess the unconditional type I error rate of the entire two stage procedure. Depending on whether the preliminary Shapiro-Wilk test was significant or not, The Mann-Whiney U test or the student’s t test was conducted for the main analysis.

For strategy I, conditional type I error rates for t test were observed to have been inflated for exponential distribution. It was however observed that conditional type I error rates were below the nominal significant level for uniform distribution. There were no significant differences observed for normal distribution. Type I Error rates for both exponential and uniform distribution were inflated for strategy II. No significant differences were again observed in normal distribution. Type I Error rates were within acceptable limits for all three distributions for the two-stage procedure. The two-stage procedure was therefore considered to be robust.

The statistical power to detect a shift in two normal distributions was also calculated for the three distributions.

It was noted that the appropriateness of a statistical test, which depends on underlying distributional assumptions, is generally not a problem if the population distribution is known in advance because if the distribution is normal, the t test can be performed, otherwise the nonparametric equivalent will be adopted. The problem arises if the population distribution is unknown, which unfortunately is the most common scenario in medical research.

**Discussions:**

Q1. Since pretesting is believed to alter the Type I Error Rates of the main test, what is the best recommendation for researchers since it is important to ensure that normality condition is satisfied before a t test? Graphical approach?

Q2. What do we say when we simulate a sample data from exponential distribution, but the results of the pretest suggest that they are normally distributed? Type I error of course. How seriously then do we take the results of the main t test?

Q3. What happens if we also perform pretest for homogeneity of variance. Do we expect Type I error rates to be more altered?

Q4. Several different recommendations about controlling Type I error rates are discussed in several articles. I see plenty of methods for conducting preliminary testing for normality discussed but not even one of them is without flaws. Some have fewer problems than others, for instance the SW test but not for all instances. I guess our goal is to, if possible, get a method that will somehow correct the failures that these previous methods have.

**Table 1a: Estimated Type I Errors of two sample t test after a passed Shapiro-Wilk test for normality.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.042 | 0.044 | 0.039 | 0.035 | 0.036 |  | **0.1** | 0.043 | 0.044 | 0.039 | 0.039 | 0.036 |
| **0.05** | 0.044 | 0.044 | 0.042 | 0.038 | 0.037 |  | **0.05** | 0.043 | 0.037 | 0.040 | 0.040 | 0.037 |
| **0.01** | 0.049 | 0.046 | 0.046 | 0.044 | 0.044 |  | **0.01** | 0.049 | 0.050 | 0.046 | 0.045 | 0.041 |
| **0.005** | 0.052 | 0.049 | 0.047 | 0.045 | 0.045 |  | **0.005** | 0.052 | 0.050 | 0.048 | 0.044 | 0.043 |
| **w/o pretest** | 0.053 | 0.048 | 0.051 | 0.052 | 0.049 |  | **w/o pretest** | 0.058 | 0.047 | 0.052 | 0.047 | 0.050 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.050 | 0.053 | 0.050 | 0.048 | 0.050 |  | **0.1** | 0.049 | 0.053 | 0.050 | 0.049 | 0.050 |
| **0.05** | 0.044 | 0.049 | 0.047 | 0.05 | 0.047 |  | **0.05** | 0.049 | 0.050 | 0.050 | 0.053 | 0.046 |
| **0.01** | 0.050 | 0.049 | 0.054 | 0.049 | 0.048 |  | **0.01** | 0.050 | 0.050 | 0.047 | 0.048 | 0.051 |
| **0.005** | 0.046 | 0.051 | 0.046 | 0.053 | 0.050 |  | **0.005** | 0.047 | 0.047 | 0.05 | 0.054 | 0.050 |
| **w/o pretest** | 0.048 | 0.051 | 0.047 | 0.049 | 0.050 |  | **w/o pretest** | 0.051 | 0.053 | 0.049 | 0.053 | 0.050 |

**Table 1b: Estimated Type I Errors of two sample Wilcoxon test after a failed Shapiro-Wilk test for normality.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.070 | 0.061 | 0.051 | 0.052 | 0.045 |  | **0.1** | 0.075 | 0.055 | 0.052 | 0.051 | 0.049 |
| **0.05** | 0.080 | 0.066 | 0.054 | 0.055 | 0.051 |  | **0.05** | 0.093 | 0.059 | 0.058 | 0.051 | 0.051 |
| **0.01** | 0.087 | 0.069 | 0.063 | 0.053 | 0.053 |  | **0.01** | 0.168 | 0.111 | 0.074 | 0.060 | 0.057 |
| **0.005** | 0.083 | 0.069 | 0.063 | 0.053 | 0.054 |  | **0.005** | 0.233 | 0.133 | 0.087 | 0.069 | 0.059 |
| **w/o pretest** | 0.082 | 0.064 | 0.061 | 0.050 | 0.053 |  | **w/o pretest** | 0.050 | 0.050 | 0.052 | 0.048 | 0.049 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.055 | 0.063 | 0.062 | 0.064 | 0.059 |  | **0.1** | 0.069 | 0.058 | 0.055 | 0.061 | 0.056 |
| **0.05** | 0.059 | 0.069 | 0.063 | 0.063 | 0.067 |  | **0.05** | 0.069 | 0.063 | 0.062 | 0.064 | 0.059 |
| **0.01** | 0.062 | 0.068 | 0.064 | 0.066 | 0.060 |  | **0.01** | 0.090 | 0.081 | 0.073 | 0.072 | 0.074 |
| **0.005** | 0.062 | 0.069 | 0.066 | 0.067 | 0.065 |  | **0.005** | 0.093 | 0.085 | 0.084 | 0.081 | 0.073 |
| **w/o pretest** | 0.063 | 0.066 | 0.069 | 0.061 | 0.062 |  | **w/o pretest** | 0.054 | 0.047 | 0.047 | 0.049 | 0.049 |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.045 | 0.048 | 0.052 | 0.052 | 0.050 |  | **0.1** | 0.050 | 0.048 | 0.051 | 0.050 | 0.051 |
| **0.05** | 0.043 | 0.050 | 0.056 | 0.049 | 0.048 |  | **0.05** | 0.052 | 0.052 | 0.050 | 0.045 | 0.053 |
| **0.01** | 0.040 | 0.052 | 0.048 | 0.050 | 0.048 |  | **0.01** | 0.055 | 0.051 | 0.049 | 0.049 | 0.047 |
| **0.005** | 0.040 | 0.047 | 0.049 | 0.050 | 0.056 |  | **0.005** | 0.060 | 0.047 | 0.052 | 0.049 | 0.048 |
| **w/o pretest** | 0.039 | 0.046 | 0.048 | 0.048 | 0.050 |  | **w/o pretest** | 0.053 | 0.050 | 0.048 | 0.050 | 0.051 |

**Table 2: Estimated Type I Error probability of the two-sample t test for samples that passed testing for normality of the residuals (strategy II) for different sample sizes and**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.091 | 0.316 | 0.000 | 0.000 | 0.000 |  | **0.1** | 0.122 | 0.398 | 0.709 | N/A | N/A |
| **0.05** | 0.092 | 0.312 | 0.000 | 0.000 | 0.000 |  | **0.05** | 0.096 | 0.317 | 0.611 | 0.839 | N/A |
| **0.01** | 0.090 | 0.314 | 0.000 | 0.000 | 0.000 |  | **0.01** | 0.072 | 0.196 | 0.421 | 0.669 | 0.859 |
| **0.005** | 0.090 | 0.314 | 0.000 | 0.000 | 0.000 |  | **0.005** | 0.064 | 0.162 | 0.347 | 0.583 | 0.792 |
| **w/o pretest** | 0.092 | 0.308 | 0.000 | 0.000 | 0.000 |  | **w/o pretest** | 0.044 | 0.048 | 0.047 | 0.049 | 0.051 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.051 | 0.050 | 0.049 | 0.053 | 0.050 |  | **0.1** | 0.049 | 0.053 | 0.048 | 0.053 | 0.048 |
| **0.05** | 0.046 | 0.051 | 0.054 | 0.048 | 0.047 |  | **0.05** | 0.051 | 0.052 | 0.051 | 0.052 | 0.053 |
| **0.01** | 0.050 | 0.048 | 0.053 | 0.052 | 0.048 |  | **0.01** | 0.049 | 0.046 | 0.048 | 0.051 | 0.049 |
| **0.005** | 0.043 | 0.052 | 0.053 | 0.048 | 0.049 |  | **0.005** | 0.045 | 0.051 | 0.050 | 0.049 | 0.051 |
| **w/o pretest** | 0.049 | 0.050 | 0.051 | 0.050 | 0.051 |  | **w/o pretest** | 0.052 | 0.051 | 0.046 | 0.051 | 0.048 |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.045 | 0.048 | 0.052 | 0.052 | 0.050 |  | **0.1** | 0.050 | 0.048 | 0.051 | 0.050 | 0.051 |
| **0.05** | 0.043 | 0.050 | 0.056 | 0.049 | 0.048 |  | **0.05** | 0.052 | 0.052 | 0.050 | 0.045 | 0.053 |
| **0.01** | 0.040 | 0.052 | 0.048 | 0.050 | 0.048 |  | **0.01** | 0.055 | 0.051 | 0.049 | 0.049 | 0.047 |
| **0.005** | 0.040 | 0.047 | 0.049 | 0.050 | 0.056 |  | **0.005** | 0.060 | 0.047 | 0.052 | 0.049 | 0.048 |
| **w/o pretest** | 0.039 | 0.046 | 0.048 | 0.048 | 0.050 |  | **w/o pretest** | 0.053 | 0.050 | 0.048 | 0.050 | 0.051 |

**Table 2: Estimated Type I Error probability of the two-sample U test for samples that passed testing for normality of the residuals (strategy II) for different sample sizes and**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.026 | 0.02 | 0.026 | 0.039 | 0.043 |  | **0.1** | 0.027 | 0.024 | 0.029 | 0.041 | 0.045 |
| **0.05** | 0.026 | 0.018 | 0.026 | 0.036 | 0.043 |  | **0.05** | 0.025 | 0.018 | 0.022 | 0.035 | 0.044 |
| **0.01** | 0.024 | 0.016 | 0.026 | 0.035 | 0.041 |  | **0.01** | 0.036 | 0.011 | 0.013 | 0.022 | 0.031 |
| **0.005** | 0.025 | 0.02 | 0.026 | 0.031 | 0.043 |  | **0.005** | 0.046 | 0.012 | 0.012 | 0.016 | 0.029 |
| **w/o pretest** | 0.024 | 0.018 | 0.023 | 0.034 | 0.043 |  | **w/o pretest** | 0.043 | 0.053 | 0.047 | 0.048 | 0.050 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.084 | 0.080 | 0.072 | 0.066 | 0.062 |  | **0.1** | 0.071 | 0.074 | 0.063 | 0.062 | 0.061 |
| **0.05** | 0.082 | 0.084 | 0.075 | 0.071 | 0.062 |  | **0.05** | 0.085 | 0.079 | 0.071 | 0.064 | 0.067 |
| **0.01** | 0.084 | 0.080 | 0.070 | 0.070 | 0.066 |  | **0.01** | 0.120 | 0.107 | 0.087 | 0.073 | 0.073 |
| **0.005** | 0.086 | 0.082 | 0.077 | 0.070 | 0.065 |  | **0.005** | 0.153 | 0.107 | 0.090 | 0.083 | 0.079 |
| **w/o pretest** | 0.082 | 0.084 | 0.072 | 0.067 | 0.064 |  | **w/o pretest** | 0.044 | 0.045 | 0.051 | 0.044 | 0.050 |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.037 | 0.046 | 0.050 | 0.047 | 0.049 |  | **0.1** | 0.037 | 0.049 | 0.047 | 0.047 | 0.050 |
| **0.05** | 0.034 | 0.045 | 0.050 | 0.048 | 0.051 |  | **0.05** | 0.034 | 0.046 | 0.048 | 0.050 | 0.050 |
| **0.01** | 0.031 | 0.050 | 0.050 | 0.052 | 0.047 |  | **0.01** | 0.034 | 0.046 | 0.045 | 0.051 | 0.050 |
| **0.005** | 0.031 | 0.045 | 0.050 | 0.052 | 0.049 |  | **0.005** | 0.036 | 0.040 | 0.047 | 0.044 | 0.048 |
| **w/o pretest** | 0.032 | 0.043 | 0.049 | 0.055 | 0.051 |  | **w/o pretest** | 0.041 | 0.054 | 0.046 | 0.044 | 0.050 |

**Table 3: Estimated Type I Error probability of the two-stage procedure (Student’s t test or Mann-Whitney’s U test depending on preliminary Shapiro Wilk test for normality) for different sample sizes and —strategy I**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.050 | 0.052 | 0.049 | 0.050 | 0.049 |  | **0.1** | 0.053 | 0.050 | 0.048 | 0.049 | 0.048 |
| **0.05** | 0.050 | 0.051 | 0.05 | 0.050 | 0.050 |  | **0.05** | 0.055 | 0.052 | 0.048 | 0.049 | 0.050 |
| **0.01** | 0.051 | 0.052 | 0.049 | 0.050 | 0.049 |  | **0.01** | 0.054 | 0.054 | 0.048 | 0.049 | 0.050 |
| **0.005** | 0.051 | 0.051 | 0.051 | 0.051 | 0.050 |  | **0.005** | 0.050 | 0.055 | 0.049 | 0.048 | 0.049 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.05 | 0.052 | 0.052 | 0.051 | 0.051 |  | **0.1** | 0.051 | 0.052 | 0.053 | 0.051 | 0.051 |
| **0.05** | 0.05 | 0.051 | 0.051 | 0.051 | 0.051 |  | **0.05** | 0.051 | 0.051 | 0.051 | 0.051 | 0.050 |
| **0.01** | 0.051 | 0.051 | 0.052 | 0.052 | 0.052 |  | **0.01** | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 |
| **0.005** | 0.050 | 0.051 | 0.052 | 0.051 | 0.051 |  | **0.005** | 0.051 | 0.050 | 0.049 | 0.050 | 0.050 |
| **Uniform Distribution** | | | | | |  | **Uniform Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.052 | 0.050 | 0.051 | 0.049 | 0.049 |  | **0.1** | 0.052 | 0.051 | 0.048 | 0.049 | 0.049 |
| **0.05** | 0.051 | 0.052 | 0.051 | 0.05 | 0.051 |  | **0.05** | 0.053 | 0.051 | 0.051 | 0.050 | 0.048 |
| **0.01** | 0.051 | 0.052 | 0.051 | 0.050 | 0.050 |  | **0.01** | 0.051 | 0.051 | 0.052 | 0.051 | 0.051 |
| **0.005** | 0.052 | 0.052 | 0.051 | 0.051 | 0.048 |  | **0.005** | 0.052 | 0.050 | 0.050 | 0.052 | 0.050 |

**Table 3: Estimated Type I Error probability of the two-stage procedure (Student’s t test or Mann-Whitney’s U test depending on preliminary Shapiro Wilk test for normality) for different sample sizes and ---Strategy II**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.050 | 0.052 | 0.049 | 0.050 | 0.049 |  | **0.1** | 0.053 | 0.050 | 0.048 | 0.049 | 0.048 |
| **0.05** | 0.050 | 0.051 | 0.050 | 0.050 | 0.050 |  | **0.05** | 0.055 | 0.052 | 0.048 | 0.049 | 0.050 |
| **0.01** | 0.051 | 0.052 | 0.049 | 0.050 | 0.049 |  | **0.01** | 0.054 | 0.054 | 0.048 | 0.049 | 0.050 |
| **0.005** | 0.051 | 0.051 | 0.051 | 0.051 | 0.050 |  | **0.005** | 0.050 | 0.055 | 0.049 | 0.048 | 0.049 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.05 | 0.052 | 0.052 | 0.051 | 0.051 |  | **0.1** | 0.051 | 0.052 | 0.053 | 0.051 | 0.051 |
| **0.05** | 0.05 | 0.051 | 0.051 | 0.051 | 0.051 |  | **0.05** | 0.051 | 0.051 | 0.051 | 0.051 | 0.050 |
| **0.01** | 0.051 | 0.051 | 0.052 | 0.052 | 0.052 |  | **0.01** | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 |
| **0.005** | 0.05 | 0.051 | 0.052 | 0.051 | 0.051 |  | **0.005** | 0.051 | 0.050 | 0.049 | 0.050 | 0.050 |
| **Uniform Distribution** | | | | | |  | **Uniform Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.052 | 0.050 | 0.051 | 0.049 | 0.049 |  | **0.1** | 0.052 | 0.051 | 0.048 | 0.049 | 0.049 |
| **0.05** | 0.051 | 0.052 | 0.051 | 0.050 | 0.051 |  | **0.05** | 0.053 | 0.051 | 0.051 | 0.050 | 0.048 |
| **0.01** | 0.051 | 0.052 | 0.051 | 0.050 | 0.050 |  | **0.01** | 0.051 | 0.051 | 0.052 | 0.051 | 0.051 |
| **0.005** | 0.052 | 0.052 | 0.051 | 0.051 | 0.048 |  | **0.005** | 0.052 | 0.050 | 0.050 | 0.052 | 0.050 |

**Table 4: Estimated power of the two -stage procedure for different sample sizes and ---strategy I**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.284 | 0.537 | 0.705 | 0.821 | 0.894 |  | **0.1** | 0.287 | 0.537 | 0.703 | 0.817 | 0.894 |
| **0.05** | 0.288 | 0.551 | 0.718 | 0.826 | 0.894 |  | **0.05** | 0.292 | 0.550 | 0.714 | 0.821 | 0.894 |
| **0.01** | 0.291 | 0.562 | 0.744 | 0.848 | 0.908 |  | **0.01** | 0.295 | 0.558 | 0.740 | 0.848 | 0.908 |
| **0.005** | 0.295 | 0.559 | 0.750 | 0.857 | 0.916 |  | **0.005** | 0.294 | 0.561 | 0.749 | 0.855 | 0.915 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.241 | 0.455 | 0.627 | 0.750 | 0.841 |  | **0.1** | 0.244 | 0.455 | 0.626 | 0.750 | 0.824 |
| **0.05** | 0.242 | 0.455 | 0.625 | 0.753 | 0.842 |  | **0.05** | 0.245 | 0.456 | 0.625 | 0.753 | 0.840 |
| **0.01** | 0.242 | 0.455 | 0.628 | 0.752 | 0.843 |  | **0.01** | 0.244 | 0.455 | 0.629 | 0.756 | 0.842 |
| **0.005** | 0.244 | 0.456 | 0.628 | 0.754 | 0.842 |  | **0.005** | 0.245 | 0.458 | 0.627 | 0.751 | 0.845 |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.240 | 0.447 | 0.618 | 0.746 | 0.836 |  | **0.1** | 0.248 | 0.446 | 0.609 | 0.743 | 0.835 |
| **0.05** | 0.250 | 0.446 | 0.615 | 0.743 | 0.835 |  | **0.05** | 0.254 | 0.451 | 0.610 | 0.744 | 0.835 |
| **0.01** | 0.247 | 0.466 | 0.619 | 0.744 | 0.837 |  | **0.01** | 0.270 | 0.467 | 0.615 | 0.744 | 0.835 |
| **0.005** | 0.245 | 0.480 | 0.623 | 0.746 | 0.837 |  | **0.005** | 0.264 | 0.482 | 0.615 | 0.743 | 0.837 |

**Table 4: Estimated power of the two -stage procedure for different sample sizes and ---strategy II**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Estimates from Our simulation | | | | | |  | Estimates from the paper | | | | | |
| Uniform Distribution | | | | | |  | Uniform Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.213 | 0.431 | 0.605 | 0.732 | 0.827 |  | **0.1** | 0.290 | 0.529 | 0.697 | 0.816 | 0.895 |
| **0.05** | 0.214 | 0.431 | 0.603 | 0.733 | 0.826 |  | **0.05** | 0.294 | 0.542 | 0.702 | 0.817 | 0.894 |
| **0.01** | 0.213 | 0.434 | 0.604 | 0.732 | 0.825 |  | **0.01** | 0.295 | 0.559 | 0.729 | 0.835 | 0.900 |
| **0.005** | 0.216 | 0.435 | 0.607 | 0.733 | 0.824 |  | **0.005** | 0.295 | 0.563 | 0.742 | 0.842 | 0.906 |
| **Normal Distribution** | | | | | |  | **Normal Distribution** | | | | | |
| **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |  | **α\_pre/n** | 10 | 20 | 30 | 40 | 50 |
| **0.1** | 0.241 | 0.455 | 0.627 | 0.750 | 0.841 |  | **0.1** | 0.244 | 0.455 | 0.626 | 0.750 | 0.824 |
| **0.05** | 0.242 | 0.455 | 0.625 | 0.753 | 0.842 |  | **0.05** | 0.245 | 0.456 | 0.625 | 0.753 | 0.840 |
| **0.01** | 0.242 | 0.455 | 0.628 | 0.752 | 0.843 |  | **0.01** | 0.244 | 0.455 | 0.629 | 0.756 | 0.842 |
| **0.005** | 0.244 | 0.456 | 0.628 | 0.754 | 0.842 |  | **0.005** | 0.245 | 0.458 | 0.627 | 0.751 | 0.845 |
| Exponential Distribution | | | | | |  | Exponential Distribution | | | | | |
| **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |  | **α\_pre/n** | **10** | **20** | **30** | **40** | **50** |
| **0.1** | 0.184 | 0.501 | 0.713 | 0.845 | 0.918 |  | **0.1** | 0.248 | 0.446 | 0.609 | 0.743 | 0.835 |
| **0.05** | 0.184 | 0.494 | 0.710 | 0.842 | 0.918 |  | **0.05** | 0.254 | 0.451 | 0.610 | 0.744 | 0.835 |
| **0.01** | 0.182 | 0.484 | 0.711 | 0.840 | 0.919 |  | **0.01** | 0.270 | 0.467 | 0.615 | 0.744 | 0.835 |
| **0.005** | 0.189 | 0.480 | 0.713 | 0.842 | 0.917 |  | **0.005** | 0.264 | 0.482 | 0.615 | 0.743 | 0.837 |